

# 4.3

## Isosceles and Equilateral Triangles

**Goal** Use properties of isosceles and equilateral triangles.

### VOCABULARY

**Legs of an isosceles triangle** The congruent sides of an isosceles triangle are called legs.

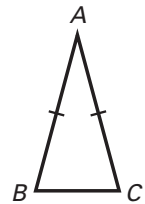
**Base of an isosceles triangle** The side of an isosceles triangle that is not a leg is called the base.

**Base angles of an isosceles triangle** The two angles at the base of an isosceles triangle are called the base angles.

### THEOREM 4.3: BASE ANGLES THEOREM

**Words** If two sides of a triangle are congruent, then the angles opposite them are congruent.

**Symbols** If  $\overline{AB} \cong \overline{AC}$ , then  $\angle B \cong \angle C$ .



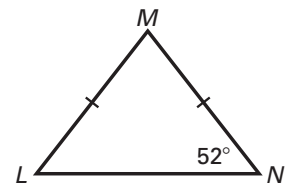
### Example 1 Use the Base Angles Theorem

Find the measure of  $\angle L$ .

#### Solution

Angle  $L$  is a base of an isosceles triangle. From the Base Angles Theorem,  $\angle L$  and  $\angle N$  have the same measure.

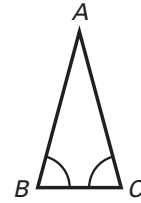
**Answer** The measure of  $\angle L$  is  $52^\circ$ .



**THEOREM 4.4: CONVERSE OF THE BASE ANGLES THEOREM**

**Words** If two angles of a triangle are congruent, then the sides opposite them are congruent.

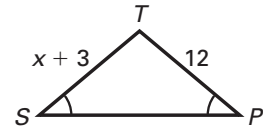
**Symbols** If  $\angle B \cong \angle C$ , then  $\overline{AC} \cong \underline{\overline{AB}}$ .

**Example 2** *Converse of the Base Angles Theorem*

Find the value of  $x$ .

**Solution**

By the Converse of the Base Angles Theorem, the legs have the same length.



$$TS = TP \quad \text{Converse of the Base Angles Theorem}$$

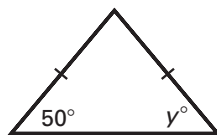
$$x + \underline{3} = \underline{12} \quad \text{Substitute for } \underline{TS} \text{ and } \underline{TP}.$$

$$x = \underline{9} \quad \text{Subtract } \underline{3} \text{ from each side.}$$

**Answer** The value of  $x$  is 9.

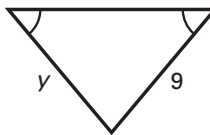
✓ **Checkpoint** Find the value of  $y$ .

1.



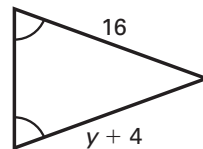
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2.



9

3.



12

**THEOREM 4.5: EQUILATERAL THEOREM**

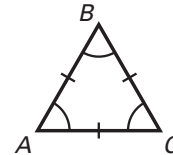
**Words** If a triangle is equilateral, then it is equiangular.

**Symbols** If  $\overline{AB} \cong \overline{AC} \cong \overline{BC}$ , then  $\angle A \cong \underline{\angle B} \cong \underline{\angle C}$ .

**THEOREM 4.6: EQUIANGULAR THEOREM**

**Words** If a triangle is equiangular, then it is equilateral.

**Symbols** If  $\angle B \cong \angle C \cong \angle A$ , then  
 $\overline{AB} \cong \underline{\overline{AC}} \cong \underline{\overline{BC}}$ .

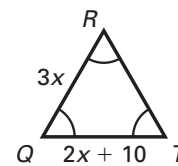
**Example 3** *Side Length of an Equiangular Triangle*

Find the length of each side of the equiangular triangle.

**Solution**

The angle marks show that  $\triangle QRT$  is equiangular.

So,  $\triangle QRT$  is also equilateral.



$$3x = \underline{2x + 10} \quad \text{Sides of an equilateral triangle are congruent.}$$

$$x = \underline{10} \quad \text{Subtract } \underline{2x} \text{ from each side.}$$

$$3(\underline{10}) = \underline{30} \quad \text{Substitute } \underline{10} \text{ for } x.$$

**Answer** Each side of  $\triangle QRT$  is 30.

**Follow-Up** Compare Example 2 and Example 3.

How are they alike? They both use congruent angles to find congruent sides, then solve for x.

How are they different? Example 2 has an isosceles triangle and Example 3 has an equilateral triangle.